Use Separate Answerscripts for each group

Undergraduate Examination 2018

Semester – III (CBCS)

Mathematics

Generic Elective Course: GEC-3

(Differential Equations and its applications)

Time: Three Hours Full Marks: 60

Questions are of value as indicated in the margin.
Notations and symbols have their usual meanings.

Group-A

Answer ***any four*** questions.

10×4=40

1. a) Construct a differential equation by elimination of arbitrary constants a and b from the equation $ax^2 + by^2 = 1$.
   Also find the order and degree of the differential equation. 2+½+½

   b) Find the general solution of the following differential equation by using the method of variation of parameters: 5

   $$\frac{d^2y}{dx^2} + y = \sec x.$$

   c) Examine for the singular solution of the differential equation

   $$9\left(\frac{dy}{dx}\right)^2 (2-y)^2 = 4(3-y).$$ 2

2. a) Show that e^x, e^{-x} and e^{2x} are three linearly independent solutions of

   $$\left(\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y\right) = 0.$$ 2+1

   Hence find the general solution.

   b) Solve the simultaneous differential equations

   $$\frac{dx}{dt} - 7x + y = 0,$$
   $$\frac{dy}{dt} - 2x - 5y = 0.$$ 4

   c) Reduce the differential equation

   $x^2\left(y - x\frac{dy}{dx}\right) = y\left(\frac{dy}{dx}\right)^2$ to Clairaut's form by the substitutions $u = x^2$ and $v = y^2$. 3

3. Find the general solution of the following differential equations:

   a) $x^2p^2 + xyp - 6y^2 = 0.$ 3

   b) $y = x + p^3.$ 4

   c) $y^2 \log y = xyp + p^2.$ 3

P.T.O.

4. a) Examine whether the equation

$$\left(a^2-2xy-y^2\right)dx-(x+y)^2\,dy=0 \text{ is exact.}$$

If it be exact then solve it. 1+1

b) Solve the following differential equations:

i) $\log\left(\frac{dy}{dx}\right)=ax+by$. 2+3+3

ii) $\left(x^2+1\right)\frac{dy}{dx}+4xy=\frac{1}{\left(x^2+1\right)^2}$.

iii) $y^3\frac{d^2y}{dx^2}=5$.

5. a) Find the orthogonal trajectories of the family of concentric circles centred at the origin. 2

b) Find the family of curves which make an angle $\pi/4$ with the hyperbolas $xy=c$. 3

c) Check whether the differential equation

$$x^2y'''+4xy''+\left(x^2+2\right)y'+3xy=2$$

is exact.

If not, find the value of m so that x^m is an integrating factor of the above differential equation. Obtain its first integral. 1+2+2

6. a) Using the substitutions $y^2=v-x$, reduce the equation $y^3\frac{dy}{dx}+x+y^2=0$ to the homogeneous form and hence solve it. 2+3

b) Find a particular integral of the differential equation

$$y'''-5y''+8y'-4y=e^{2x}+e^x+\sin x.$$ 2

c) Convert the differential equation

$$(1+2x)^2\frac{d^2y}{dx^2}-6(1+2x)\frac{dy}{dx}+16y=8(1+2x)^2$$

to the differential equation with constant coefficients and hence solve it. 1+2

Group-B

Answer ***any two*** questions.

10×2=20

1. a) Find the partial differential equation of all spheres of unit radius with centres on the $y-z$ plane. Obtain its degree and order. State the nature of the equation. 3+1+1

b) Find the partial differential equations of all planes which are at a constant distance from the origin. Find the partial differential equation by elimination of arbitrary function f from $z=x+y+f(xy)$. 3+2

2. a) Find the surface which intersects the surfaces of the one-parametric system $z=3cxy\left(x^2+y^2\right)$ orthogonally and which passes through the hyperbola $x^2-y^2=a^2$, $z=0$. 5

b) Find the general integral of the partial differential equation $2x\left(y+z^2\right)p+y\left(2y+z^2\right)q=z^3$. Write a short note on geometrical interpretation of the partial differential equation

$$P(x,y,z)z_x+Q(x,y,z)z_y=R(x,y,z).$$ 3+2

3. a) Show that $z=ax+by+a^2+b^2$ is a complete integral of the partial differential equation $z=px+qy+p^2+q^2$. Hence obtain the singular integral, if any, and particular integral assuming $a=b$. 2+1½+1½

b) Show that the partial differential equation $p^2x^2+q^2y^2=z^2$ can be transformed to the form $f(p,q)=0$ by using suitable transformation. Hence solve the equation using Charpit's rule. 3+2

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